Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

Henrik Jensen
Department of Economics
University of Copenhagen

MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 16 EXAM, 2014

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) There are never limits on how much seigniorage a public sector can extract in a model with a conventional money demand function.

A False. In the curriculum we have seen examples showing that inflationary finance has revenue limits. As inflation goes up, money demand goes down thus eroding the "tax base" for seigniorage. Hence, one can have an inflation Laffer-curve relationship implying that there is a maximum level of seigniorage that can be extracted.

(ii) In the basic New-Keynesian model, shocks to productivity A_t, in the aggregate production function $Y_t = A_t N_t$ (where Y_t is output and N_t is labor), pose an inflation–output trade off for the monetary policymaker.

A False. Fluctuations in productivity cause fluctuations in the natural rate of output. Appropriate monetary policy can in the basic model make actual output track the natural rate of output perfectly, thereby keeping the output gap closed, and thus avoiding any inflation. In consequence, there is no inflation-output policy trade off.

(iii) In the Poole (1970) model, the case for a interest-rate operating procedure gets weaker, all things equal, when a relationship between the broad money supply and base money is introduced.

A False. In the basic Poole model, the case for an interest rate operating procedure is strengthened the more volatility there is on the money market. Introducing a relationship between broad and narrow money does not affect this. On the contrary if there is exogenous noise between the money concepts (e.g., a stochastic money multiplier as seen in the curriculum), the case for an interest-rate operating procedure becomes stronger.

QUESTION 2:

Assume a flex-price, closed economy in discrete time, where households maximize

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t, n_t\right)$$

with

$$u\left(c_t, m_t, n_t\right) \equiv \frac{\left(c_t m_t\right)^{1-\Phi}}{1-\Phi} + \frac{\left(1-n_t\right)^{1-\eta}}{1-\eta}, \qquad \Phi > 0, \quad \eta > 0,$$

subject to the budget constraints

$$f\left(k_{t-1}, n_t\right) + \tau_t + \left(1-\delta\right) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{1}$$

where c_t is consumption, m_t is real money balances at the end of period t, n_t is fraction of time spent working, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers from the government, $0 < \delta < 1$ is the depreciation rate of capital, and π_t is the inflation rate. The function f is defined as

$$f\left(k_{t-1}, n_t\right) = k_{t-1}^{\alpha} n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

(i) Discuss why money may enter the utility function, and describe (1) in detail.

A Assuming money provides utility is a short cut. It can be a short cut for the value of the broad liquidity services money provide, or more specifically the saved leisure from not having to engage in barter. Equation (1) shows how a household can use its available resources in a period. Resources are on the left-hand side and are total income (wage and capital income), real monetary transfers from the government and the real value of asset holdings net of depreciation. The assets are the capital stock net of physical depreciation (δ), and real money holdings net of inflation. The right-hand-side is what resources are used for: consumption and the “new” asset holdings.

(ii) Derive the relevant first-order conditions for optimal choices of c, m, k, and n subject to (1) and the definition

$$a_t \equiv \tau_t + \frac{1}{1+\pi_t} m_{t-1}$$

For this purpose, use the value function $V(a_t, k_{t-1}) = \max\{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\}$. Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into

$$u_m(c_t, m_t, n_t) + \frac{\beta}{1+\pi_{t+1}} u_c(c_{t+1}, m_{t+1}, n_{t+1}) = u_c(c_t, m_t, n_t) \tag{2}$$

$$u_c(c_t, m_t, n_t) = \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \tag{3}$$

$$-u_n(c_t, m_t, n_t) = u_c(c_t, m_t, n_t) f_n(k_{t-1}, n_t), \tag{4}$$

where $R_t \equiv f_k(k_t, n_{t+1}) + 1 - \delta$ is the gross real interest rate, which equals $(1+i_t)/(1+\pi_{t+1})$, with i_t being the nominal interest rate.

A We set up the value function

$$V(a_t, k_{t-1}) = \max\{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\},$$

and note that (1) and the definition of a_t give

$$\begin{aligned} k_t &= -c_t - m_t + f(k_{t-1}, n_t) + a_t + (1-\delta) k_{t-1}, \\ a_{t+1} &= \tau_{t+1} + \frac{1}{1+\pi_{t+1}} m_t. \end{aligned}$$

We substitute these into $V(a_{t+1}, k_t)$ to make an unconstrained problem where we maximize over c, m and n. We get the following first-order conditions:

$$\begin{aligned} u_c(c_t, m_t, n_t) - \beta V_k(a_{t+1}, k_t) &= 0, && (*) \\ u_m(c_t, m_t, n_t) + \beta V_a(a_{t+1}, k_t) \frac{1}{1+\pi_{t+1}} - \beta V_k(a_{t+1}, k_t) &= 0, && (**) \\ u_n(c_t, m_t, n_t) + \beta V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t) &= 0. && (***) \end{aligned}$$

Equation (*) shows that households choose consumption to balance the marginal utility against the marginal loss in terms of less next-period capital. Equation (**) shows that households choose money holdings to balance the marginal gains (consisting of the marginal utility per se plus the future marginal value of money) against the loss in terms of less next-period capital. Finally, equation (***) shows that households choose labour supply so as to balance the marginal

utility loss per se against the marginal gain of more next-period capital times the marginal product of labor.
To complete the characterization of optimum, we differentiate V with respect to a_t and k_{t-1} taking into account that any effects through c_t, m_t and n_t can be ignored due to the envelope theorem. We therefore find

$$\begin{aligned} V_a(a_t, k_{t-1}) &= \beta V_k(a_{t+1}, k_t) && (****) \\ V_k(a_t, k_{t-1}) &= \beta V_k(a_{t+1}, k_t)(f_k(k_{t-1}, n_t) + 1 - \delta) && (*****) \end{aligned}$$

Using (*) in (***) to eliminate $V_k(a_{t+1}, k_t)$ readily gives (4). Forwarding (*****) one period gives

$$\begin{aligned} V_k(a_{t+1}, k_t) &= \beta V_k(a_{t+2}, k_{t+1})(f_k(k_t, n_{t+1}) + 1 - \delta) \\ V_k(a_{t+1}, k_t) &= \beta V_k(a_{t+2}, k_{t+1}) R_t \end{aligned}$$

which by use of (*) to eliminate $V_k(a_{t+1}, k_t)$ and $V_k(a_{t+2}, k_{t+1})$ gives (3). Forwarding (****) one period and use it in (**) to eliminate $V_a(a_{t+1}, k_t)$ gives

$$u_m(c_t, m_t, n_t) + \frac{\beta^2}{1+\pi_{t+1}} V_k(a_{t+2}, k_{t+1}) - \beta V_k(a_{t+1}, k_t) = 0.$$

Then, using (*) to eliminate $V_k(a_{t+1}, k_t)$ and $V_k(a_{t+2}, k_{t+1})$ gives (2).

(iii) With the specific functional forms for u and f, examine the properties of the steady state using (2), (3), and (4) together with the national account identity $c^{ss} = k^{ss^{\alpha}} n^{ss^{1-\alpha}} - \delta k^{ss}$. Assess under which circumstances the model exhibits superneutrality or not. For that purpose focus on whether changes in m_t (induced by changes in inflation and nominal interest rates) have real effects. Explain intuitively the transmission mechanism, which leads to potential non-superneutrality.

A Equations (2), (3) and (4) in the steady state become (using u and f)

$$\begin{aligned} (c^{ss})^{1-\Phi}(m^{ss})^{-\Phi} + \frac{\beta}{1+\pi^{ss}}(c^{ss})^{-\Phi}(m^{ss})^{1-\Phi} &= (c^{ss})^{-\Phi}(m^{ss})^{1-\Phi}, \\ (c^{ss})/(m^{ss}) + \frac{\beta}{(1+\pi^{ss})} &= 1 \\ (c^{ss})/(m^{ss}) + \frac{1}{R^{ss}(1+\pi^{ss})} &= 1 \end{aligned}$$

and

$$\begin{aligned} 1 &= \beta\left(\alpha (k^{ss})^{\alpha-1}(n^{ss})^{1-\alpha} + 1 - \delta\right), \\ (1-n^{ss})^{-\eta} &= (c^{ss})^{-\Phi}(m^{ss})^{1-\Phi}(1-\alpha)(k^{ss})^{\alpha}(n^{ss})^{-\alpha} \end{aligned}$$

Put differently,

$$
\begin{aligned}
\left(c^{ss}\right)/\left(m^{ss}\right) &= \frac{i^{ss}}{1+i^{ss}}, \\
1 &= \beta\left(\alpha\left(k^{ss}/n^{ss}\right)^{\alpha-1}+1-\delta\right), \\
\left(1-n^{ss}\right)^{-\eta} &= \left(c^{ss}\right)^{-\Phi}\left(m^{ss}\right)^{1-\Phi}(1-\alpha)\left(k^{ss}/n^{ss}\right)^{\alpha}
\end{aligned}
$$

So, k^{ss}/n^{ss} and the real interest rate are independent of inflation. Using the resource constraint we find

$$c^{ss}/n^{ss} = \left(k^{ss}/n^{ss}\right)^{\alpha} - \delta\left(k^{ss}/n^{ss}\right).$$

Hence, the ratio c^{ss}/n^{ss} is independent as well. Use this in the money demand equation to find

$$\frac{c^{ss}}{n^{ss}} = \frac{m^{ss}}{n^{ss}}\frac{i^{ss}}{1+i^{ss}}$$

Now multiply by $\left(n^{ss}\right)^{\Phi}$ in the labor supply schedule:

$$\left(n^{ss}\right)^{\Phi}\left(1-n^{ss}\right)^{-\eta} = \left(c^{ss}/n^{ss}\right)^{-\Phi}\left(m^{ss}\right)^{1-\Phi}(1-\alpha)\left(k^{ss}/n^{ss}\right)^{\alpha}$$

As the left-hand-side is an increasing function of n^{ss}, we see that higher m^{ss} will increase (decrease) labor supply when Φ is lower (higher) than one. The reason is that in the case of $\Phi > 1$, the marginal utility of consumption decreases with higher m^{ss} leading consumers to substitute towards leisure, and thus supply less labor. This could happen when inflation and the nominal interest rate decreases (which tend to increase real money holdings).
Showing the above algebraically is difficult, so providing the intuitive economic arguments is given high credit.

QUESTION 3:

Consider the following log-linear model of a closed economy:

$$
\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(\widehat{i}_t - \mathrm{E}_t\pi_{t+1}\right), \qquad \sigma > 0, && (1)\\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0<\beta<1, \quad \kappa>0, && (2)\\
\widehat{i}_t &= \phi\pi_t, \qquad \phi > 1, && (3)
\end{aligned}
$$

where x_t is the output gap, $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, and π_t is goods-price inflation, e_t is a mean-zero "cost-push" shock without serial correlation. E_t is the rational-expectations operator conditional upon all information up to and including period t.

(i) Explain the economics of (1) and (2) with focus on the underlying micro-economic foundations. What does (3) represent? Explain.

A Equation (1) is the dynamic IS curve, which is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $\widehat{\imath}_t -$ $\mathrm{E}_t \{\pi_{t+1}\}$, make consumers increase future consumption relative to current. Equation (2), the New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be in effect for some periods. Equation (3) is a simple specification for how monetary policy, in terms of nominal interest rate setting, is determined. It is a simple Taylor-type rule where the nominal interest rate in increased (more than one-for-one) when inflation increases, which secures uniqueness of equilibrium in the model.

(ii) Derive the solutions for x_t and π_t. [Hint: Conjecture that the solutions are linear functions of e_t, and use the method of undetermined coefficients.] Comment on the role of the policy parameter ϕ in terms of the output gap's and inflation's dependence on e_t, and discuss whether the parameter can be chosen such that the output gap and inflation are stabilized completely.

A Using the hint, we conjecture that the solutions take the following forms

$$\begin{aligned} x_t &= Ae_t, \\ \pi_t &= Be_t, \end{aligned}$$

where A, B are the undetermined coefficients to be identified. Forwarding the conjectures one period, and taking period-t expectations give

$$\begin{aligned} \mathrm{E}_t x_{t+1} &= A\mathrm{E}_t e_{t+1} = 0, \\ \mathrm{E}_t \pi_{t+1} &= B\mathrm{E}_t e_{t+1} = 0, \end{aligned}$$

where $\mathrm{E}_t e_{t+1} = 0$ follows from the definition of the process for e_t. We then insert the conjecture, the associated expectations and the Taylor rule into equations (1) and (2) and obtain

$$\begin{aligned} Ae_t &= -\sigma^{-1}\phi Be_t, \\ Be_t &= \kappa Ae_t + e_t. \end{aligned}$$

This verifies the form of the conjectures (it is consistent with the model), and since it holds for all e_t, we identify A and B from

$$\begin{aligned} A &= -\sigma^{-1}\phi B, \\ B &= \kappa A + 1. \end{aligned}$$

This gives

$$\begin{aligned} A &= -\frac{\sigma^{-1}\phi}{1+\kappa\sigma^{-1}\phi}, \\ B &= \frac{1}{1+\kappa\sigma^{-1}\phi}, \end{aligned}$$

and therefore

$$\begin{aligned} x_t &= -\frac{\sigma^{-1}\phi}{1+\kappa\sigma^{-1}\phi} e_t, \\ \pi_t &= \frac{1}{1+\kappa\sigma^{-1}\phi} e_t. \end{aligned}$$

From these solutions one sees that with $\phi > 1$, the inflationary impact of a positive shock ($e_t > 0$) is dampened, as the central bank raises the real interest rate. This comes at the cost of a negative output gap. There is thus a trade off present in monetary policy, and one can see that full stabilization of both inflation and the output gap is not possible. If, e.g., $\phi \to \infty$ the central bank will be able to stabilize inflation completely, but the output gap will become $-\kappa^{-1}e_t$ ($\lim_{\phi\to\infty} -\frac{\sigma^{-1}\phi}{1+\kappa\sigma^{-1}\phi} = -\kappa^{-1}$). So, the higher is ϕ the more stable is inflation, and the less stable is the output gap.

Assume that a welfare-relevant loss function can be written as

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right), \qquad \lambda > 0. \tag{4}$$

(iii) Derive the welfare-optimal values of x_t and π_t under discretionary policymaking [hence, equation (3) no longer applies]. For this purpose, treat x_t as the policy instrument, and show that the relevant first-order condition for optimal policy together with (2) yield the difference equation

$$\pi_t = \frac{\lambda\beta}{\kappa^2+\lambda}\mathrm{E}_t\pi_{t+1} + \frac{1}{1+\kappa^2/\lambda}e_t. \tag{5}$$

Find the unique solutions for π_t and x_t, and discuss whether commitment of the central bank can improve on policymaking.

A Under discretion, expectations are taken as given, so we have a sequence of one-period minimization problems:

$$\begin{aligned} \min_{x_t} L(\pi_t, x_t) &\equiv \frac{1}{2}\left(\pi_t^2 + \lambda x_t^2\right) \\ \text{s.t.} \quad \pi_t &= \kappa x_t + v_t \\ v_t &\equiv \beta \mathrm{E}_t \pi_{t+1} + e_t \end{aligned}$$

The relevant first-order condition is

$$\lambda x_t = -\kappa \pi_t. \tag{*}$$

Insert this back into (2) to eliminate x_t:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \frac{\kappa^2}{\lambda}\pi_t + e_t,$$

which is rewritten as

$$\pi_t \left(1 + \kappa^2/\lambda\right) = \beta \mathrm{E}_t \pi_{t+1} + e_t,$$

which can readily be written as (5).
Since, $\lambda\beta/\left(\lambda + \kappa^2\right) < 1$, (5) has a unique stationary solution. This solution is found by conjecturing

$$\pi_t = Ce_t.$$

This implies

$$\mathrm{E}_t \pi_{t+1} = 0,$$

and we therefore immediately get from (5) that $C = 1/\left(1 + \kappa^2/\lambda\right)$, and thus

$$\pi_t = \frac{1}{1 + \kappa^2/\lambda} e_t.$$

Combining this with the first-order condition (*) gives

$$x_t = -\frac{\kappa/\lambda}{1 + \kappa^2/\lambda} e_t.$$

We see that under discretion, the temporary shock e_t has only temporary effects on inflation and output. If the central bank was able to commit to a future path of policies, it could in this framework use this ability to affect inflation expectations. This would provide another channel for inflation stabilization. For example, if a temporary positive shock hits the economy, the central bank can improve over discretion by promising a contractionary policy that continues into the future. This will put downward pressure on inflation expectations and help reducing current inflation. Hence, the *history dependence*, or *inertia*, which characterize commitment improves policy outcomes. (Another way to frame this is to say that the inflation-output trade off is improved under commitment, as a given output contraction reduces current inflation by more, if the contraction is expected to persist.)